Law of Sines

Law of Sines:

In any △ABC with angles A, B, C opposite sides *a*, b, c respectively, the following equation is true:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Solving Triangles (AAS, ASA)

Two angles and a side of a triangle, in any order, determine the size and shape of a triangle completely.

Ex.

Solve $\triangle ABC$ given that $\angle A = 36°, \angle B = 48°$, and a = 8.

We know that $\angle C = 180° - 36° - 48° = 96°$.

Using the Law of Sines we can find b and c.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{and} \quad \frac{\sin A}{a} = \frac{\sin C}{c}$$

So, $$\frac{\sin 36°}{8} = \frac{\sin 48°}{b} \quad \text{and} \quad \frac{\sin 36°}{8} = \frac{\sin 96°}{c}$$

$$b = \frac{8\sin 48^\circ}{\sin 36^\circ} \qquad c = \frac{8\sin 96^\circ}{\sin 36^\circ}$$

$$b \approx 10.115 \qquad c \approx 13.536$$

The Ambiguous Case (Side, Side, Angle)

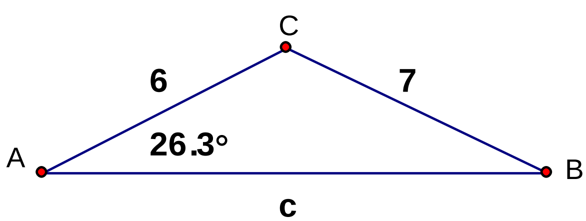
If the angle is between the two sides, one triangle is uniquely determined, but if the angle is opposite one of the sides then, one, two or zero triangles are determined.

Solving a Triangle given Two Sides and an Angle

Solve $\triangle ABC$ given that *a* = 7, b = 6, and $\angle A = 26.3^\circ$.

By drawing a triangle we can determine that this is not the ambiguous case.

Start by solving for the acute angle B, using the Law of Sines.



$$\frac{\sin 26.3^\circ}{7} = \frac{\sin B}{6}$$

$$\sin B = \frac{6\sin 26.3^\circ}{7}$$

$$B = \sin^{-1}\left(\frac{6\sin 26.3^\circ}{7}\right) = 22.3^\circ$$

Now we can find angle C, and then use the Law of Sines to find c.

When solving a triangle you must analyze the data to determine whether there is a solution or not.

- When A < 90°

If *a* > b there is one unique solution.

If *a* < b there are three possibilities.

One solution if *a* = b sin A

No solution is *a* < b sin A

Two solutions if b > *a* > b sin A

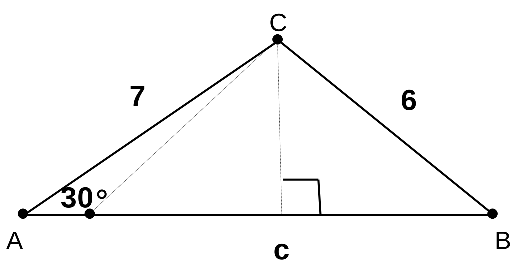
- When A ≥ 90° there are two possibilities.

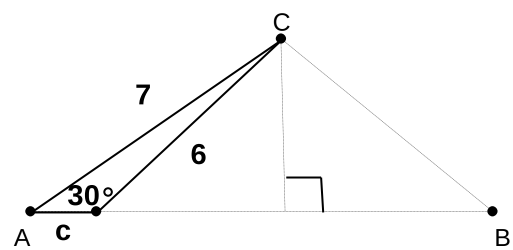
No solution *a* ≤ b One solution *a* > b

Handling the Ambiguous Case:

Solve $\triangle ABC$ given that *a* = 6, b = 7, and $\angle A = 30°$.

Two triangles are possible with the given information.





Using the Law of Sines we can start by finding $\angle B$.

$$\frac{\sin 30°}{6}=\frac{\sin B}{7} \qquad \sin B=\frac{7\sin 30°}{6} \qquad B=35.7°$$

This is the acute angle shown in the first triangle. The calculator will not give an obtuse angle. We will continue to solve the triangle with acute angle B.

$\angle C=114.3°$ and using the Law of Sines we can find c.

Now we will consider the possibility that $\angle B$is obtuse. From the second triangle we can see that $\angle B$= $180°-35.7°=144.3°$.

By subtraction the acute $\angle C=5.7°$. Now we can use the Law of Sines to find c.